**Question 1**

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| T | T | T | F |
| T | F | T | F |
| F | T | T | T |
| F | F | F | F |

Since is neither always true nor always false, it is a contingency.

(Liu 2020a)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| T | T | F | T | F |
| T | F | T | F | F |
| F | T | T | F | F |
| F | F | T | F | F |

Since is always false, it is a contradiction.

(Liu 2020a)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |
| T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | T |
| T | F | T | F | T | F | T | T |
| T | F | F | F | T | F | F | T |
| F | T | T | T | T | T | T | T |
| F | T | F | T | F | F | T | T |
| F | F | T | T | T | T | T | T |
| F | F | F | T | T | T | T | T |

Since is always true, it is a tautology.

(Liu 2020a)

**Question 2**

(Liu 2020a)

(Gray 2014)

**Question 3**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |
| T | T | T | T | T | T | T | T | F |
| T | T | F | T | T | T | T | T | T |
| T | F | T | T | F | T | T | F | T |
| T | F | F | T | F | T | F | T | T |
| F | T | T | T | T | T | T | T | F |
| F | T | F | T | T | F | T | T | T |
| F | F | T | F | T | T | T | F | T |
| F | F | F | F | T | F | F | T | T |

From the table, we notice that all 5 of the propositions to be can be made simultaneously true (when is true, is true and is false)

**Question 4**

“A compound proposition is satisfiable if there is an assignment of truth values to the variables in the compound proposition that makes the statement form true (Rosen 2007, 30). Moreover, “A compound proposition that is always true, no matter what the truth values of the propositions that occur in it, is called a *tautology*.” (Rosen 2007, 21). Therefore, in accordance with the above definitions, a compound proposition which is deemed to be a tautology is also satisfiable.

With the aforementioned in mind, in order to determine whether a given compound proposition is a tautology, such a program could, firstly, check whether the proposition is satisfiable. Thereafter, if the proposition is deemed unsatisfiable, it is by definition, also not a tautology and, therefore, program execution should stop. Otherwise, if the proposition is satisfiable, program execution should continue, checking if there is an assignment of truth values that makes the compound proposition false. If there is such an assignment, the proposition is merely satisfiable and program execution should stop. Otherwise, the proposition is tautological and by definition, satisfiable and since its tautology has been determined, program execution should also stop.

**Question 5**

**Question 6**

1. Some student likes Chinese and all students like Mexican
2. Some cuisine is liked by Monica or Jay
3. Amongst all students, there is a cuisine that two students don’t like
4. For some student, there is a cuisine that this student likes and this student doesn’t like any other cuisine
5. For some student, there is another student where amongst all cuisines, both students like a particular cuisine or neither students like that cuisine
6. Amongst all students, there is a cuisine that two students like or neither students like

(Rosen 2007)

**Question 7**

(Rosen 2007)

**Question 8**

1. Let “ is mortal”, where the domain of is all men

;

;

Valid, Universal Instantiation

(Liu 2020b)

1. Let “ is a man”, where the domain of is all men

Let “ is an island”, where the domain of is all islands

Valid, Universal Instantiation and Modus Tollens

(Liu 2020b)

1. Let “ is an action movie”

Let “Chandler likes movie ”

The domain of is all movies

Invalid, Fallacy of Converse

(Liu 2020b)

1. Let “student knows how to write programs in Python”

Let “student x can get a high-paying job”,

The domain of is all students in Computing

Valid, Modus Ponens and Existential Generalisation

(Liu 2020b)

1. Let “car is a convertible”

Let “car is fun to drive”

The domain of is all cars

Invalid, Fallacy of Inverse

(Liu 2020b)

1. Let = “Superman is able to prevent evil”

Let = “Superman is willing to prevent evil”

Let = “Superman prevents evil”

Let = “Superman is impotent”

Let = “Superman is malevolent”

Let = “Superman exists”

First, let’s denote the givens:

Second, let’s put it all together:

1. , from 1, 4
2. , from 2, 3, 6 (Resolution)
3. from 5, 6 (Resolution)

Since Superman does not exist”, this argument is valid.

(Rosen 2007)

**Question 9**

1. Show that

* Assume
* Assume

**Proof**

where and

the sum of two odd integers is even

(Maths and Stats 2017)

1. i.e. Show that

* Assume

**Proof**

where and

by definition

Since , it follows that

(maths gotserved 2014)

1. i.e. Show that

* Assume

**Proof**

where and and

Since by definition

(Kahn Academy 2013)

1. i.e. Show that , where and

**Proof**

|  |  |
| --- | --- |
| is true | is true |
| is true | is true |

is true

1. i.e. Show that where denotes

**Proof**

|  |  |  |
| --- | --- | --- |
| is true | is true | is true |
| is true | is true |  |

Since for and for , is true

**Question 10**

1. is the base step of .

is true

1. The inductive hypothesis is the implication
2. You need to prove that is true
3. For , let denote the statement

**Base Step**

View Question 10b

**Inductive Step**

For an arbitrary , assuming that is true, it remains to prove that,

, given below, holds.

Starting with the LHS of ,

we see that the RHS of follows.

By completing the inductive step, we have proven that is true

By mathematical induction, we have also proven that for any, the statement is true

i.e. We have proven that implication is true

(Rosen 2007)

**Question 11**

For , let denote the statement

**Base Step**

**Inductive Step**

For an arbitrary , assuming that is true, it remains to prove that , given below, holds.

Starting with the LHS of ,

we see that the RHS of follows.

By completing the inductive step, we have proven that is true

By mathematical induction, we have also proven that for any, the statement is true

i.e. We have proven that the implication is true

(Rosen 2007)

**Question 12**

For , let denote the statement “Postage of cents can be formed using just 3-cent stamps and 5-cent stamps”.

**Base Steps**

is true

is true

is true

From the above, we notice above that where and .

**Inductive Step**

For an arbitrary , assuming that , , , , are true, show that .

Since we seek to show that is true, we can use in our proof, which was proven to be true by inductive hypothesis as

Our base cases of , and can be used to form any when a multiple of 3 is added.

e.g.

For , postage of -cents can be formed using just 3-cent stamps and 5-cent stamps.

(Rosen 2007)

**References**

Liu, Wanquan. 2020a. "Lecture 1. Propositional Logics." Slides.

Liu, Wanquan. 2020b. "Lecture 3. Methods of Proof." Slides

Gray, Kailee. 2014. *Proving logical equivalence involving the biconditional*. YouTube video, 21:02. https://www.youtube.com/watch?v=FYr5dX0h6K8&ab\_channel=KaileeGray

Rosen, Kenneth. 2007. *Discrete Mathematics and Its Applications*. 6 ed. New York: Mcgraw-Hill.

Maths and Stats. 2017. *The sum of two Odd numbers are Even*. YouTube video, 4:21. https://www.youtube.com/watch?v=aBOgPLgpIfE&ab\_channel=MathsandStats

maths gotserved. 2014. *indirect proof 3 Prove that if n^3+5 is odd then n is even contradiction contraposition*. YouTube video, 16:37. https://www.youtube.com/watch?v=pFDEBLSUxaU&ab\_channel=mathsgotserved

Kahn Academy. 2013. *Proof that sum of rational and irrational is irrational | Algebra I | Khan Academy*. https://www.youtube.com/watch?v=pPM72fPwIjw&ab\_channel=KhanAcademy